

Orange County Math Circle New Year MATHCOUNTS Scrimmage

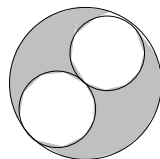
Sprint Round

Name:

School:

Grade:

1. Compute: $4 + 5 - 7 \times 9 \div 3$.
2. The first place trophy of the tournament is 15 inches tall. 1 inch is approximately 2.54 centimeters. How tall is the trophy in centimeters? Express your answer as a decimal to the nearest tenth.
3. Jared has nine coins in his pocket. They all look alike, but one coin is counterfeit and weighs less than the others. What is the least number of weighings on a balance scale needed to guarantee that the counterfeit coin is found?
4. The sum of three numbers is 81 and their ratio is $3 : 7 : 17$. What is the value of the smallest number?
5. July 4, 1903, was a Tuesday. On what day of the week was July 4, 1904?
6. What is the median of the composite integers that are greater than 20 and less than 35?
7. From a bag of coins, $\frac{1}{3}$ were given to Mary, $\frac{1}{5}$ to Norm, $\frac{1}{6}$ to Anna, and $\frac{1}{4}$ to Brian. The six left were given to Troy. How many coins were originally in the bag?
8. Alex wants to provide waters and cookies for 95 students who participate in the Orange County Math Circle New Year Invitational. Water bottles are sold in packages of eight; cookies are sold in packages of ten. If he purchased the minimum number of packages of each to guarantee at least one water bottle and one cookie for each participants, how many more water bottles than cookies did he buy?
9. Two tangent congruent circles are circumscribed by a larger circle. The diameter of the larger circle is 24 cm. How many square centimeters are in the area of the shaded region? Express your answer in terms of π .



10. A four-digit number is created by using each of the digits 4, 5, 8, and 9 exactly once. What is the probability that the number will be a multiple of 4? Express your answer as a common fraction.
11. Two different circles and a triangles lie in the same plane. What is the greatest number of points that can belong to at least two of the three figures?
12. In a savings account, \$5000 is invested at an annual rate of 9% simple interest. How many dollars would have to be invested at 7.5% simple interest to earn an equal amount of interest?
13. The measures of the three angles of a triangle form an arithmetic sequence. If the smallest angle measures 45° , what is the number of degrees in measure of the largest angle?

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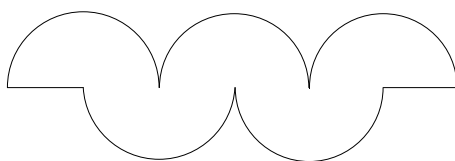
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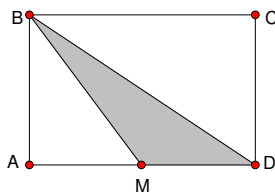
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14. One year ago, the number of years in Jane's age was a perfect square, and one year from now, her age will be a perfect cube. How many years old is Jane?
15. A 12-hour digital clock shows only the hours and the minutes. During what fraction of the time will there be at least one 1 in the display? Express your answer as a common fraction.
16. Each curve in the logo is a semicircle with a radius of 8 cm. What is the area of the logo? Express your answer in terms of π ?



17. What is the unit digit of 2^{48} ?
18. A car holds exactly six people, but only two of these six people can drive the car. What is the number of ways that the six people can be seated in the car on a drive?
19. A slug climbs ten inches in ten minutes. It then rests two minutes. It continues climbing at a constant rate and rests for two minutes after climbing ten minutes. How many minutes will it take the slug to reach the top of a twenty-foot tower?
20. The point M is the midpoint of segment BE . What percent of the area of rectangle $ABCD$ is shaded?



21. What is the least positive integer with exactly 10 factors?
22. When several thieves tried to divide a sum of money by giving \$4 to each thief, one thief received nothing. When each thief took \$3, they had \$1 left over. What is the sum of the number of dollars and the number of thieves?
23. Richard took the elevator to the 8th floor, then down 5 floors, up 10 floors, and finally down 1 floor. If those moves left Richard on the middle floor of the building, how many floors are in the building?
24. Given that $f(x) = x^{-1} + \frac{x^{-1}}{1+x^{-1}}$, what is $f(f(-2))$? Express your answer as a common fraction.
25. Rachel subtracted two positive numbers and the difference was 12. Britt multiplied the same two numbers and the product was 540. What is the sum of the numbers?

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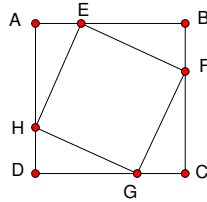
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26. The vertices of square $EFGH$ lie on the edge of square $ABCD$. $\frac{AE}{EB} = \frac{1}{2}$. What is the ratio of the area of square $EFGH$ to the area of square $ABCD$?



27. Two students set their digital watches to 10 : 00. One watch runs one minute per hour too slow, and the other watch runs 2 minutes per hour too fast. What time will the slow watch show when it is exactly one hour behind the fast watch?
28. Al can do a particular job in 9 days. Betty can do 20% more work each day than Al. How many days does it take Betty to do the same job? Express your answer as a decimal to the nearest tenth.
29. Julian and Vino working together can paint a room in 11 hours. Vino and Ted working together can paint the same room in 10 hours, and Ted and Julian working together can paint the room in 11 hours. What is the number of hours that it will take Julian, Vino and Ted working together to paint 31 identical rooms?
30. Rectangle $ABCD$ and parallelogram $EFHG$ are shown. The area of isosceles triangle EAF is 8 square inches, $BH = 6$ in, $CD = 20$ in, and segment EK is perpendicular to segment FG . The distance from E to K is equal to 88 inches divided by some number. What is that number in simplest radical form?

